Fictionalism

According to the previous proposals, the statements of ordinary mathematics are true when suitably, i.e., nominalistically, interpreted. The nominalistic account of mathematics that will now be discussed holds that all existential mathematical statements are false simply because there are no mathematical entities. (For the same reason all universal mathematical statements will be trivially true.)

Fictionalism holds that mathematical theories are like fiction stories such as fairy tales and novels. Mathematical theories describe fictional entities, in the same way that literary fiction describes fictional characters. This position was first articulated in the introductory chapter of (Field 1989), and has in recent years been gaining in popularity.

This crude description of the fictionalist position immediately opens up the question what sort of entities fictional entities are. This appears to be a deep metaphysical ontological problem. One way to avoid this question altogether is to deny that there exist fictional entities. Mathematical theories should be viewed as invitations to participate in games of pretence, in which we act as if certain mathematical entities exist. Pretence or make-believe operators shield their propositional objects from existential exportation (Leng 2010).

Anyway, as said above, on the fictionalist view, a mathematical theory isn't literally true. Nonetheless, mathematics is used to get truths across. So we must *subtract* something from what is literally said when we assert a physical theory that involves mathematics, if we want to get at the truth. But this requires a *theory* of how this subtraction of content works. Such a theory has been developed in (Yablo, 2014).

If the fictionalist thesis is correct, then one demand that must be imposed on mathematical theories is surely consistency. Yet Field adds to this a second requirement: mathematics must be *conservative* over natural science. This means, roughly, that whenever a statement of an empirical theory can be derived using mathematics, it can in principle also be derived without using any mathematical theories. If this were not the case, then an indispensability argument could be played out against fictionalism. Whether mathematics is in fact conservative over physics, for instance, is currently a matter of controversy. Shapiro has formulated an incompleteness argument that intends to refute Field's claim (Shapiro 1983).

If there are indeed no mathematical (fictional) entities, as one form of fictionalism has it, then Benacerraf's epistemological problem does not arise. Fictionalism then shares this advantage over most forms of platonism with nominalistic reconstructions of mathematics. But the appeal to pretence operators entails that the logical form of mathematical sentences then differs somewhat from their surface form. If there are fictional objects, then the surface form of mathematical sentences can be taken to coincide with their logical form. But if they exist as abstract entities, then Benacerraf's epistemological problem reappears.

Whether Benacerraf's identification problem is solved is not completely clear. In general, fictionalism is a non-reductionist account. Whether an entity in one mathematical theory is identical with an entity that occurs in another theory is usually left indeterminate by mathematical "stories". Yet Burgess has rightly emphasized that mathematics differs from literary fiction in the fact that fictional characters are usually confined to one work of fiction, whereas the same mathematical entities turn up in diverse mathematical theories (Burgess 2004). After all, entities with the same *name* (such as π)π) turn up in different theories. Perhaps the fictionalist can maintain that when mathematicians develop a new theory in which an "old" mathematical entity occurs, the entity in question is made more precise. More determinate properties are ascribed to it than before, and this is all right as long as overall consistency is maintained.

The canonical objection to formalism seems also applicable to fictionalism. The fictionalists should find some explanation of the fact that extending a mathematical theory in one way, is often considered preferable over continuing it in a another way that is incompatible with the first. There is often at least an appearance that there is a right way to extend a mathematical theory.

Source: https://plato.stanford.edu/entries/philosophy-mathematics/